Stability Analysis

We are familiar with the classic stability criterion for the advective and diffusive terms:

Or

And so long the pressure is solved for exactly, there is no stability requirement for the pressure term. However, suppose the pressure at the subsequent step is not fully resolved, then there exists a similar criteria for the pressure term. This document investigates this restriction. The NS equations are

Discretizing this using explicit Euler, we have

Where

Defining

We may write

Taking the divergence of this equation to try to make the divergence at the next step to be zero results in

At the next step we may write

And so long the pressure is solved for exactly, there is no stability requirement for the pressure term. However, suppose the pressure at the subsequent step is not fully resolved. Let's assume that there is some associated error at step n, say . We may write our pressure at step n as

Then, substituting this back into the correction step we have